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Quantum magnetotransport in tilted magnetic fields: exact results for parabolic wells

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Abstract. The energy level structure of a quasi-two-dimensional electron system in a parabolic quantum well with the magnetic field tilted with respect to the sample plane was investigated. Based on this model and on the linear response theory the analytical expressions for the magnetoconductivity tensor were derived. Owing to the anisotropy induced by the tilted field, two diagonal components of conductivity are no longer equivalent. In high-mobility samples, precisely quantized Hall plateau develop in the off-diagonal component of conductivity regardless of field orientation.

1. Introduction

The magnetotransport properties of the quasi-two-dimensional electron systems confined in a narrow quantum well have attracted attention for many years. The main attention has been paid to the special configuration with the magnetic field perpendicular to the confinement plane. In this case the electron motion can be separated into an electric contribution governed by the confining potential and a magnetic contribution leading to the formation of Landau levels. The separation of variables in the one-electron Hamiltonian of the system makes it possible to describe the electronic transport in terms of a two-dimensional (2D) gas, i.e. to neglect the transverse motion completely. For any other configuration of the magnetic field this separation is in general not possible and the electron energy structure is more complicated. A review of work devoted to this problem up to 1980 was given in [1]. Two different theoretical approaches have appeared. First, perturbation theory and numerical methods may be applied in the case of realistic models of quantum wells and electron energy dispersion laws. Then the experimental data can be interpreted semiquantitatively for a narrow range of parameters. The second choice is to investigate simple, analytically solvable models which can explain qualitatively the gross features of the data obtained for a broad spectrum of various experimental arrangements. An important step in this direction was made by Maan [2] and Merlin [3] who found analytically the subband structure for the case of electrons in a magnetic field of arbitrary orientation and for parabolic quantum wells. The aim of this publication is to extend their work and also to derive analytic expressions for components of the conductivity tensor for the same model.

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2. General theory

A system of non-interacting electrons which are mobile in the x–y plane and confined in the z direction is considered. A uniform magnetic field $\boldsymbol{B}$ is applied to the system at an arbitrary angle with respect to z. The corresponding one-electron Hamiltonian has the form

$$H = (1/2m)[\boldsymbol{p} - (e/c)\boldsymbol{A}]^2 + V(\boldsymbol{r}) \tag{1}$$

where e and m are the electron charge and mass, respectively, c is the velocity of light and $\boldsymbol{A}$ is the vector potential ($\boldsymbol{B} = \mathrm{curl}\,\boldsymbol{A}$). Both the electron confinement and the elastic scattering of electrons by impurities are described by the potential $V(\boldsymbol{r})$.

Let us assume that an electric current $\boldsymbol{I}$ is linearly related to the applied electric field $\boldsymbol{E}$ through Ohm's law

$$\boldsymbol{I} = \boldsymbol{\sigma}\boldsymbol{E} \tag{2}$$

where $\boldsymbol{\sigma}$ is the conductivity tensor.

Under the above assumptions the components of the conductivity tensor are given by

$$\sigma_{ij}(T) = -\int \frac{\mathrm{d}\rho(\eta)}{\mathrm{d}\eta}\,\sigma_{ij}(\eta, 0)\,\mathrm{d}\eta \tag{3}$$

where i, j stand for x, y; $\rho(\eta)$ is the equilibrium Fermi–Dirac distribution function and $\sigma_{ij}(\mu, 0)$ denotes components of the conductivity tensor at temperature $T = 0$ and μ is the Fermi energy.

A suitable form for the general expression of zero-temperature conductivity was derived in [4] based on the linear response theory. The diagonal components can be written as

$$\sigma_{ii} = \pi\hbar e^2\,\mathrm{Tr}[v_i\delta(\mu - H)v_i\delta(\mu - H)] \tag{4}$$

and the non-diagonal components read

$$\sigma_{ij} = (e^2/2)\,\mathrm{Tr}[\delta(\mu - H)(r_i v_j - r_j v_i)] + \mathrm{i}\hbar(e^2/2)\,\mathrm{Tr}[v_i G^+(\mu)v_j\delta(\mu - H) - v_i\delta(\mu - H)v_j G^-(\mu)] \tag{5}$$

where the Green functions are defined by

$$G^{\pm}(\mu) = (\mu - H \pm \mathrm{i}0)^{-1} \qquad \delta(\mu - H) = -(1/2\pi\mathrm{i})[G^+(\mu) - G^-(\mu)] \tag{6}$$

and the velocity operators are given by the commutation relations

$$v_i = (1/\mathrm{i}\hbar)[r_i, H] = (1/m)[p_i - (e/c)A_i] \tag{7}$$

where r_i stands for x and y.

Note that the form of σ_{ij} given by equation (5) involves only electrons with energy equal to the Fermi energy μ. The first term in (5) is closely related to the magnetic moment of the system:

$$M_z = \frac{e}{2c}\int_{-\infty}^{\mu} \mathrm{Tr}[\delta(\eta - H)(xv_y - yv_x)]\,\mathrm{d}\eta \tag{8}$$

and is equal to $ec\,\partial M_z/\partial\mu$. This expression is very sensitive to the compensation between the bulk diamagnetic currents and the edge currents in the sample. Replacing this term by the less sensitive expression $ec\, \partial N/\partial B_z$, as discussed for example in [5], we can avoid this problem and the standard model of the 2D gas may be used with the influence of edges on the electron structure completely neglected. Thus, the final form of σ_{ij} used throughout this paper is

$$\sigma_{ij} = ec\, \partial N/\partial B_z + i\hbar(e^2/2)\, \mathrm{Tr}[v_i G^+(\mu) v_j \delta(\mu - H) - v_i \delta(\mu - H) v_j G^-(\mu)] \tag{9}$$

where the number N of electrons is given by

$$N = \int_{-\infty}^{\mu} \mathrm{Tr}[\delta(\eta - H)]\, \mathrm{d}\eta. \tag{10}$$

3. Level structure in a parabolic well

In the following we shall use the representation with the basis formed by eigenstates $|\alpha\rangle$ of the Hamiltonian H_0 which describes a free electron in the magnetic field $\boldsymbol{B} = (0, B_y, B_z)$ and in a parabolic potential well $V(z) = \frac{1}{2}m\Omega^2 z^2$. This model was investigated previously by Maan [2] and Merlin [3]; here we use a slightly different notation. The choice of the vector potential gauge is close to that in Maan's work: $\boldsymbol{A} = (-B_z y + B_y z, 0, 0) \equiv B(-y \cos\varphi + z \sin\varphi, 0, 0)$ where φ is the angle between the z axis and the direction of the magnetic field $\boldsymbol{B}$. The Schrödinger equation corresponding to this Hamiltonian has the form

$$\{(1/2m)[p_x - (e/c)(B_y z - B_z y)]^2 + (1/2m)p_y^2 + (1/2m)p_z^2 + (m\Omega^2 z^2)/2\}\Psi = E\Psi. \tag{11}$$

The momentum component p_x is a constant of motion and we can therefore write $\Psi = \exp(\mathrm{i}kx)\, u(y, z)$. Equation (11) represents two coupled harmonic oscillators and it can be diagonalized by a rotation $y_1 = y\cos\beta + z\sin\beta$, $z_1 = -y\sin\beta + z\cos\beta$ with the angle of rotation β given by

$$\tan 2\beta = -(\omega_c^2 \sin 2\varphi)/(\omega_c^2 \cos 2\varphi - \Omega^2) \tag{12}$$

where $\omega_c = |e|B/mc$ is the cyclotron frequency.

The eigenenergies become

$$E_{nm} = \hbar\omega_1(n + \tfrac{1}{2}) + \hbar\omega_2(m + \tfrac{1}{2}) \qquad nm = 0, 1 \ldots \tag{13}$$

with eigenfrequencies ω_1 and ω_2 determined by

$$\omega_{1,2} = \{\tfrac{1}{2}[\omega_c^2 + \Omega^2 \mp (\omega_c^4 + \Omega^4 - 2\omega_c^2\Omega^2 \cos 2\varphi)^{1/2}]\}^{1/2}. \tag{14}$$

The frequency ω_1 corresponds to the harmonic motion of an electron along the y_1 axis, and ω_2 to the motion along the z_1 axis. The eigenfunctions $|\alpha\rangle$ of equation (11) can be denoted by the good quantum numbers $|\alpha\rangle = |nmk\rangle$, the eigenenergies E_{nm} are degenerate in the quantum number k. The degeneracy of each level is equal to $|e|B_z/hc$, i.e. the filling factor is determined only by the field component perpendicular to the confinement plane.

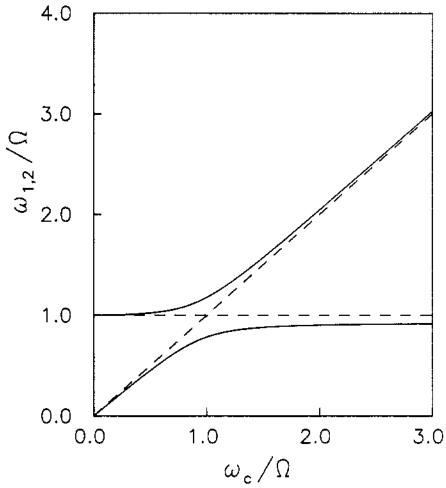

Figure 1. Magnetic field dependence of the eigenfrequencies of the hybrid magnetoelectric modes: ---, $\varphi = 0°$; ——, $\varphi = 22.5°$.

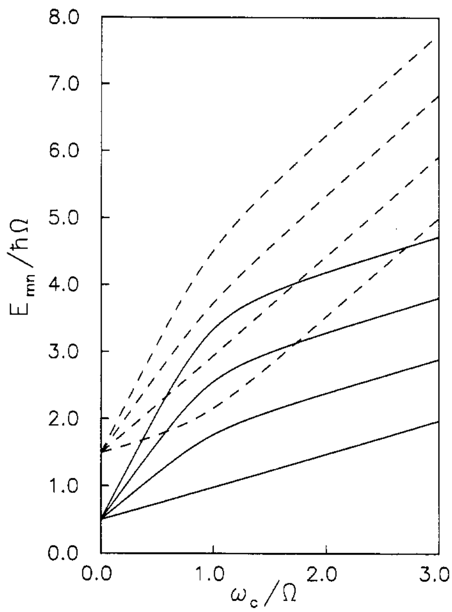

Figure 2. Eigenergies against magnetic field fan diagram corresponding to $\varphi = 22.5°$: ——, levels $m = 0, n = 0, 1, 2, 3$; ---, levels $m = 1, n = 0, 1, 2, 3$.

The magnetic field dependences of the eigenfrequencies ω_1, ω_2 and of several selected eigenenergies are shown in figures 1 and 2. For small magnetic fields, $\omega_1 \to \omega_c \cos\varphi$ and $\omega_2 \to \Omega$. Thus ω_1 corresponds to the cyclotron motion caused by z component of the magnetic field and ω_2 to the harmonic motion due to the parabolic well. In the high-field limit the roles of ω_1 and ω_2 are reversed and $\omega_1 \to \Omega \cos\varphi$, $\omega_2 \to \omega_c$. The hybrid magnetoelectric modes are formed for medium fields $\omega_c \sim \Omega$. The eigenfrequency ω_1 varies from ω_c for $\varphi = 0°$ to 0 for $\varphi = 90°$, and ω_2 goes from Ω to $(\Omega^2 + \omega_c^2)^{1/2}$ in the same range of angles.

In disordered systems the energy levels will have a finite width. We assume the case of randomly distributed zero-range impurities. Then the configurational averaging procedure for the Green functions yields the homogeneous effective medium described by a self-energy $\Sigma = \Delta - i\Gamma$. The resulting averaged resolvent is diagonal in the α representation and we can define its real and imaginary parts by

$$\langle\alpha|G^+(\eta)|\alpha'\rangle = [R_{nm}(\eta) + iF_{nm}(\eta)]\delta_{\alpha\alpha'} \tag{15}$$

where

$$F_{nm}(\eta) = -\Gamma/(X^2 + \Gamma^2) \qquad R_{nm}(\eta) = X/(X^2 + \Gamma^2) \qquad X = \eta - E_{nm} - \Delta(\eta). \tag{16}$$

The number of electrons for the system with broadened levels can be written as

$$N = \frac{|e|B_z}{hc}\left(-\frac{1}{\pi}\right)\sum_{nm}\int_{-\infty}^{\mu} F_{nm}(\eta)\,d\eta. \tag{17}$$

If we consider a constant number N of electrons, this equation implicitly defines the Fermi energy μ as a function of magnetic field $\boldsymbol{B}$ and of an angle φ.

4. Conductivity tensor

The configurationally averaged conductivity involves the quantity $\langle GvG\rangle$. For our zero-range potentials we shall assume that $\langle GvG\rangle = \langle G\rangle v\langle G\rangle$ and then only the averaged resolvent introduced in the previous paragraph enters the expression for conductivity components.

In addition to the resolvents, the velocity components v_x and v_y appear in equations (4) and (9). Their matrix elements in the α representation read

$$\langle\alpha|v_x|\alpha'\rangle = -\omega_c\cos(\beta+\varphi)\langle\alpha|y_1-y_0|\alpha'\rangle + \omega_c\sin(\beta+\varphi)\langle\alpha|z_1-z_0|\alpha'\rangle \tag{18}$$

$$\langle\alpha|v_y|\alpha'\rangle = -(\mathrm{i}\hbar/m)[\cos\beta\langle\alpha|\partial/\partial y_1|\alpha'\rangle + \sin\beta\langle\alpha|\partial/\partial z_1|\alpha'\rangle] \tag{19}$$

where $y_0 = (\hbar k\cos\beta)/(m\omega_c\cos\varphi)$ and $z_0 = -(\hbar k\sin\beta)/(m\omega_c\cos\varphi)$ are the centres of cyclotron motion in the y_1 and z_1 directions, respectively. In (18) and (19) we took into account the fact that $|\alpha\rangle$ are defined as functions of variables y_1 and z_1 and transformed v_x and v_y to the same coordinate system. Using these expressions and identities (not presented here) relating the angles β and φ to the frequencies ω_1, ω_2, Ω and ω_c, we arrive, after some manipulation, at the formulae

$$\sigma_{xx} = [(\omega_2^2-\omega_c^2)/(\omega_2^2-\omega_1^2)]\sigma_1 + [(\omega_1^2-\omega_c^2)/(\omega_1^2-\omega_2^2)]\sigma_2 \tag{20}$$

$$\sigma_{yy} = (\omega_2^2/\Omega^2)[(\omega_1^2-\Omega^2)/(\omega_1^2-\omega_2^2)]\sigma_1 + (\omega_1^2/\Omega^2)[(\omega_2^2-\Omega^2)/(\omega_2^2-\omega_1^2)]\sigma_2. \tag{21}$$

In the above expressions the quantities

$$\sigma_1 = \frac{e^2\hbar}{\pi m}\sum_\alpha \hbar\omega_1(n+1)F_{nm}F_{n+1m} \tag{22}$$

$$\sigma_2 = \frac{e^2\hbar}{\pi m}\sum_\alpha \hbar\omega_2(m+1)F_{nm}F_{nm+1} \tag{23}$$

are formally equal to the transverse magnetoconductivities of two 2D systems. The transport in the x–y_1 plane is described by σ_1; σ_2 corresponds to the electron motion in the x–z_1 plane. In accord with equations (20) and (21) the conductivities σ_{xx} and σ_{yy} are the weighted combinations of σ_1 and σ_2. Note that in general $\sigma_{xx} \simeq \sigma_{yy}$. An exception is the case $\varphi \to 0$ when $\sigma_{yy} \to \sigma_{xx} \to \sigma_1$ for $\omega_c \ll \Omega$ and $\sigma_{yy} \to \sigma_{xx} \to \sigma_2$ for $\omega_c \gg \Omega$.

The evaluation of σ_{xy} and σ_{yx} from equation (9) is complicated by terms containing the real parts of G^+ and G^-. We can avoid this difficulty using the identity

$$\tfrac{1}{2}(R_{nm}F_{n+1m} - R_{n+1m}F_{nm}) = -(\hbar\omega_1/2\Gamma)F_{nm}F_{n+1m} \tag{24}$$

and a similar expression containing ω_2 instead of ω_1. Then the off-diagonal components of conductivity can be written as

$$\sigma_{xy} = -\sigma_{yx} = ec\,\partial N/\partial B_z - \omega_1\tau(\omega_2/\Omega)[(\omega_1^2-\Omega^2)/(\omega_1^2-\omega_2^2)]\sigma_1$$
$$+ \omega_2\tau(\omega_1/\Omega)[(\omega_2^2-\Omega^2)/(\omega_2^2-\omega_1^2)]^2 \tag{25}$$

where $\tau = \hbar/2\Gamma$ denotes the relaxation time. The number N of electrons is given by (17) and its derivative with respect to B_z is evaluated assuming that μ is a constant. The terms containing σ_1 and σ_2 have an analogy in the quasiclassical description of the system. The expression $ec\,\partial N/\partial B_z$ has no quasiclassical analogy and plays an important role in the dissipationless electron transport in the quantum Hall regime.

The two conductivities σ_1 and σ_2 can be given a form more appropriate for comparison with previous results and allowing deeper physical insight:

$$\sigma_1 = \frac{e^2}{h}\frac{\hbar\omega_1\hbar\omega_2}{\hbar\Omega}\left(-\frac{1}{\pi}\right)\sum_{nm}\frac{1}{1+\omega_1^2\tau^2}[\omega_1\tau(n+\tfrac{1}{2})F_{nm} + R_{nm}] \tag{26}$$

$$\sigma_2 = \frac{e^2}{h}\frac{\hbar\omega_1\hbar\omega_2}{\hbar\Omega}\left(-\frac{1}{\pi}\right)\sum_{nm}\frac{1}{1+\omega_2^2\tau^2}[\omega_2\tau(m+\tfrac{1}{2})F_{nm} + R_{nm}]. \tag{27}$$

For $\omega_1\tau > 1$ and $\omega_2\tau > 1$ the terms containing F_{nm} dominate and oscillations of conductivities are determined by oscillations of the density of states. The positions of maxima coincide with the energy levels but their amplitudes are modified by different prefactors for σ_1 and σ_2. The structure of equations (26) and (27) reminds one of the well known Drude–Zener formulae; in fact, equations (26) and (27) reduce to the Drude–Zener formulae in the quasiclassical limit when oscillatory behaviour can be neglected.

The assumptions $\omega_1\tau \gg 1$ and $\omega_2\tau \gg 1$ lead to the expressions

$$\sigma_1 = \frac{e^2}{h}\frac{\omega_2}{\Omega}2\Gamma\sum_{nm}(n+\tfrac{1}{2})\left(-\frac{1}{\pi}\right)F_{nm} \tag{28}$$

$$\sigma_2 = \frac{e^2}{h}\frac{\omega_1}{\Omega}2\Gamma\sum_{nm}(m+\tfrac{1}{2})\left(-\frac{1}{\pi}\right)F_{nm} \tag{29}$$

similar to those obtained in the Ando [6] model of magnetoconductivity in 2D systems. They are often used for interpretation of experimental data together with a semiempirical expression

$$(-1/\pi)F_{nm} = (\tfrac{1}{2}\pi\Gamma_0^2)^{-1/2}\exp[(\mu - \mathrm{E}_{nm})/\Gamma_0]^2 \tag{30}$$

where

$$\Gamma_0^2 = (2/\pi)(|e|B_z/mc)(\hbar/\tau_0) \tag{31}$$

and τ_0 is the relaxation time at zero magnetic field (see, e.g., [1]).

5. Simple model calculation

Usually σ_{xx} and σ_{xy} are measured as functions of B for several selected angles, using the samples with a fixed number of carriers [7]. No attempt is made here to fit our results to any available experimental data. Instead we use a very simple model to illustrate the general features of the magnetic field and angle dependence of conductivity components. We evaluate them in the weak-scattering limit $\Gamma \ll k_BT$ introduced in [8]. The temperature-dependent conductivity components are calculated from equation (3) and

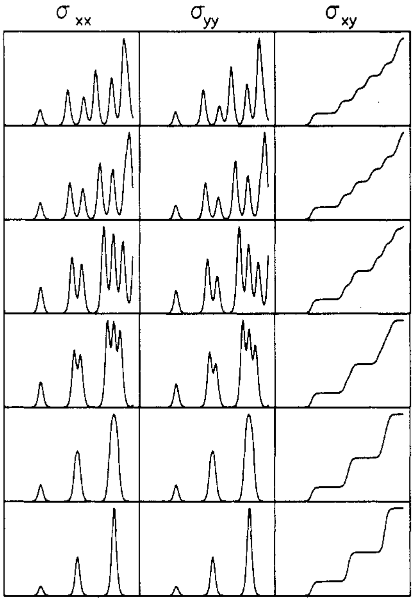

Figure 3. Magnetoconductivity components as functions of $E/\hbar\Omega$ for φ ranging from 0° for the lowest row to 25° for the top row in steps of 5°. $E/\hbar\Omega$ varies from 0 to 3.5 on the horizontal axis of each window. The case of degenerate levels $\omega_c = \Omega$ is shown: $k_B T = 0.04\hbar\Omega$.

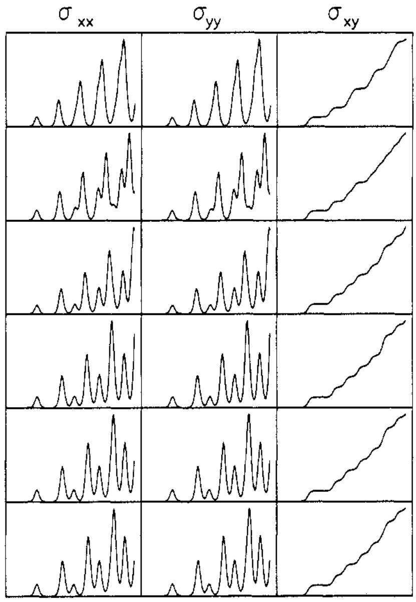

Figure 4. Magnetoconductivity components as functions of $E/\hbar\Omega$ presented for a more general level structure corresponding to $\omega_c = 0.7\Omega$. The other parameters are the same as in figure 3.

subsequently replaced by the lowest-order terms of their power expansion with respect to $\Gamma/k_B T$. In this approximation,

$$\sigma_1 = \frac{e^2}{h}\frac{\omega_2}{\Omega}\frac{\Gamma}{2k_B T}\sum_{nm}(n+\tfrac{1}{2})\frac{1}{\cosh^2(\frac{1}{2}\mu_{nm})} \tag{32}$$

$$\sigma_2 = \frac{e^2}{h}\frac{\omega_1}{\Omega}\frac{\Gamma}{2k_B T}\sum_{nm}(m+\tfrac{1}{2})\frac{1}{\cosh^2(\frac{1}{2}\mu_{nm})} \tag{33}$$

$$ec\frac{\partial N}{\partial B_z} = \frac{e^2}{h}\sum_{nm}\tfrac{1}{2}[1-\tanh(\tfrac{1}{2}\mu_{nm})] \tag{34}$$

where $\mu_{nm} = (E_{nm} - \mu)/k_B T$. Using these expressions, σ_{xx}, σ_{yy} and σ_{xy} were evaluated and shown in figures 3 and 4 as functions of $\mu/\hbar\omega_c$ for angles φ ranging from 0 to 25° in steps of 5°. The lowest row corresponds to $\varphi = 0°$ and the top row to $\varphi = 25°$. Two sets of parameters were employed. First, we considered the special case of degenerate levels $\omega_c = \Omega$ for $\varphi = 0°$. Removal of the degeneracy by tilting the magnetic field is clearly seen in figure 3. It manifests itself as a broadening of degenerate levels for small angles and their splitting for somewhat larger angles. For large angles the states which originated in different degenerate levels overlap. The difference between σ_{xx} and σ_{yy} is observable but small. This is partly due to the way of presenting the curves which neglects their absolute magnitudes. The σ_{xy} curves exhibit Hall plateaux whenever σ_{xx} and σ_{yy} reach zero. Figure 4 presents a more general case $\omega_c = 0.7\Omega$ to demonstrate the sensitivity of conductivity to the magnetic field magnitude. The curves for $\varphi = 0°$ differ substantially from those in figure 3 and their variation as functions of φ is less pronounced. A systematic shift of peak positions towards the lowest energies with increasing φ is mainly due to ω_1 which is close to $\omega_c \cos\varphi$ while ω_2 is less sensitive to the variation in φ.

6. Conclusions

On the basis of the linear response theory we have derived the general formulae for the magnetoconductivity tensor of a quasi-2D electron system in the presence of a tilted magnetic field assuming elastic scattering of independent electrons.

For the parabolic confining potential, two hybrid magnetoelectric modes develop in the case of a tilted field instead of purely electric and magnetic contributions to the electron motion known for systems with a perpendicular magnetic field arrangement. A 2D conductivity formula can be given for each mode and the resulting conductivity is a combination of two 2D conductivities.

The oscillations of magnetoconductivity have maxima at the new hybrid level positions and their amplitudes are modified by prefactors strongly dependent on the field angle and strength. Two diagonal components of conductivity, σ_{xx} and σ_{yy}, are not equivalent and $\sigma_{xx} = -\sigma_{yx}$ exhibits precisely quantized Hall plateaux e^2/h for an arbitrary magnetic field orientation.

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